Abstract

During the current Covid-19 pandemic, the learning system in all educational institution is shifted from face to face to online learning, including universities. This study aims to describe the results of the mathematical communication skills of class B students of Mathematics Education from Ahmad Dahlan University for the 2019/2020 academic year during online learning during the Covid-19 pandemic and to examine students' mistakes in solving differential calculus problems regarding the minimum, maximum material. This study employed descriptive qualitative research. The research instrument used is a test and interview guide. The data are collected using tests and interviews. The research subjects were selected by stratified random sampling technique so that 6 students are classified into high, medium and low categories. The questions are in the form of a description of one question with six questions. The results showed that students' mathematical communication skills on indicators (1) restating information was done appropriately, (2) writing down ideas or problem-solving steps clearly and precisely, some were not thorough, (3) presenting information using graphic images again. There are still things that they couldn't understand and (4) Writing down the answers according to what is known and are asked using mathematical formulas, some are still wrong because they are not accurate.

Keywords: Mathematical Analysis, Mathematical Communication, Online Learning, Covid-19
INTRODUCTION

The current Covid-19 pandemic demands a social distancing as an effort to avoid and break the spreading of the coronavirus, which means that there is a decrease in the frequency of meeting each other physically (Mona, 2020, p. 123). Social distancing includes a minimum distance of about two meters to interact with other people, use masks, maintain cleanliness such as washing hands or using hand sanitizer, and Work From Home (WFH) as a substitute for working in an office (De Vos, 2020, p. 1; Yanti et al., 2020, p. 4). Social distancing is not only applied in the workplace but also in the educational system. Social distancing is applied in all educational institutions in Indonesia and around the world (Aji, 2020, p. 396; Setiawan, 2020, p. 29; Yanti et al., 2020, p. 4). According to the Circular of the Minister of Education and Culture of the Republic of Indonesia No. 3 of 2020 concerning the Prevention of Covid-19 in the Education Unit that all higher education in Indonesia conducts learning from home, including universities. All tertiary institutions shifted the learning system, which initially took place face-to-face, to online learning.

The online learning process is applied to all students in universities in Indonesia, especially in learning mathematics. Their thinking patterns in learning mathematics certainly involved high-level cognitive abilities such as analytical skills, synthesis abilities, evaluation abilities to improve and develop mathematical power (Achir et al., 2017, p. 80; Fadillah & Jamilah, 2016, p. 107; Jamilah & Septianawaty, 2017, p. 57). Mathematical power is the ability to solve mathematical problems and also problems in real life, carry out exploration, make hypotheses, be able to state logical reasons, be able to communicate through mathematics, and connect ideas in mathematics with other fields (Jamilah & Septianawaty, 2017, p. 57; Nartani et al., 2015, p. 284; NCTM, 2000). Thus, there are several mathematical powers that can be developed in the mathematics learning process.

One of the mathematical powers that are very important to be developed in the learning process is mathematical communication skills because these abilities are not skills that already exist in students (Ariawan & Nufus, 2017, p. 84; Purwati & Nugroho, 2017, p. 127). Mathematical communication skills are closely related to cognition because the information received will be processed in the brain so that different
mathematical ideas appear to solve problems (Achir et al., 2017, p. 80). As stated by Novianti (Novianti, 2017, p. 54) that the symbolic language of mathematics can lead to clear, precise, and accurate communication. Several studies relate mathematical communication skills with problem-solving, which provide information that both are interrelated and support to improve students' mathematical power (Achir et al., 2017, p. 80; Jamilah & Septianawaty, 2017, p. 57; Khoiriyah, 2016, p. 203; Novianti, 2017, p. 54).

It can also be said that mathematical communication skills are closely related to problem-solving.

Mathematical communication skills, as an important aspect of mathematics to solve problems, are supported by various opinions about the importance of it (Jamilah & Septianawaty, 2017, p. 69; Johar et al., 2018, p. 204; Khoiriyah, 2016; Yuniar et al., 2018, p. 60). Among them, according to Barrody (Barrody, 1993), there are at least two important reasons for developing mathematical communication skills, including (1) Mathematics as a tool to find patterns, solve problems, make decisions, and communicate various ideas in a clear, accurate, and precise manner; (2) mathematics as a social activity in the learning process shown by the interaction between students and the communication between teachers and students. Furthermore, mathematical communication skills are important to sharpen thinking and the ability to observe various mathematical material relations and organize mathematical thinking (Nartani et al., 2015, p. 284; Triana et al., 2019, p. 1). In fact, (Umar, 2012b, p. 8) revealed that mathematical communication skills are one of the hearts in the mathematics learning process. Meaning that if there is no communication in the mathematics learning process, then there will be no knowledge sharing between them (Purwati & Nugroho, 2017, p. 128). This is because when students are challenged to convey their arguments to others orally or in writing, of course, there will be a learning process to communicate and convince others.

During online learning, mathematical communication skills are needed to be developed. This is a challenge to be able to develop students' mathematical communication skills well. With these skills, students can organize mathematical thinking orally and in writing, then convey responses to their friends (Umar, 2012a, p. 1).

Undergraduate students of Ahmad Dahlan University majoring in Mathematics Education Program are prepared and equipped to become prospective mathematics
teachers at the Junior High School (SMP) / equivalent and Senior High School (SMA) / Vocational High School (SMK) / equivalent. The mathematical communication skills will help them to convey mathematical knowledge to their future students professionally. Those skills are a way to share ideas, clarify an understanding, construct meaning, and define ideas over a long period of time (Novianti, 2017, p. 54; Rohid et al., 2019, p. 21). Mathematical communication skills are the ability to represent mathematical problems or ideas through concrete objects, pictures, graphs, or tables, and can apply mathematical symbols and notations (Paridjo & Waluya, 2017, p. 61; Tiffany et al., 2017, p. 2161; Triana et al., 2019, p. 2). Meanwhile, indicators of mathematical communication skills according to NCTM (2000) include: (1) Communicating mathematical thinking to others in a structured, logical, and clear manner; (2) Using mathematical language to express mathematical ideas correctly; (3) Organize and ensure mathematical thinking through communication; (4) Analyse and evaluate mathematical thinking and strategies used by others.

The achievement of mathematical communication skills will not be separated from the curriculum (Jamilah & Septianawaty, 2017, p. 58). In research conducted by (Khoiriyah, 2016, p. 208) it showed that two out of three students are able to convey ideas to solve problems about the volume of rotating objects through images clearly and precisely. Furthermore, research conducted by (Jamilah & Septianawaty, 2017, p. 70) showed that students' mathematical communication skills get an overall average of 70.16, starting to write mathematical ideas, interpreting, and evaluating mathematical ideas, and mastering the meaning of mathematical notations and symbols. Then, research conducted by (Syarifah, 2017, p. 53) also proved that the level of mathematical communication skills in solving problems in integral calculus material is in a good category, namely 58.8%.

These studies are focused on calculus courses. One of the calculus courses that students must take, especially in the Mathematics Education Program, is Differential Calculus. Usually, this course is used as a prerequisite for taking other calculus courses such as integral calculus and advanced calculus, depending on the policies of the respective universities (Fatimah & Yerizon, 2019, p. 80). The Differential Calculus course's importance requires students to be able to master every material, especially the minimum-maximum material. Researchers only limit the analysis of prospective mathematics teachers' mathematical communication skills on the minimum-maximum
material through online learning during the Covid-19 pandemic. Through this research, it is expected that preliminary data on mathematical communication skills can be obtained. Thus, researchers can make innovations as an effort to improve and develop mathematical communication skills on the minimum-maximum material.

**METHOD**

This study employed descriptive research. It was conducted online in class B of Mathematics Education Program in Faculty of Teacher Training and Education, Ahmad Dahlan University (FKIP UAD). This research was conducted in the Final Semester Examination (UAS) in the differential calculus course. About 25 students of class F semester III Mathematics Education Program, Ahmad Dahlan University (FKIP UAD) academic year 2019/2020 were grouped into three categories, including the high, medium and low category.

The procedures in this study are as follows: (1) Conducting a test; (2) Checking and assessing the results of students' answers; (3) Analyzing the results of students' answers based on indicators of mathematical communication skills; and (5) Conducting interviews with the subjects to obtain more in-depth and accurate information.

The instruments in this study were written tests and interview guidelines. The test questions in this study were used to determine their mathematical communication skills related to the questions on the maximum and minimum material. The questions were in the form of a description of six questions. The interview guidelines used in this study were unstructured.

The data were collected through interviews, documentation, and tests. The interview technique in this study was applied to explore honest, accurate, and in-depth information from the students. While the test was used to determine students' mathematical communication skills on the maximum and minimum material in the differential calculus course.

The data analysis technique was adapted from the Miles and Huberman model consisting of: (1) Data reduction; (2) Presentation of data; and (3) Drawing conclusions. Data reduction means making a summary, selecting the main and important things. Presentation of data means presenting data in the form of a brief description, chart, table,
graph, or diagram. Drawing conclusions or verification through data collection methods, such as conducting interviews. The model is shown in Figure 1 below.

![Figure 1. Miles & Huberman Model](image)

Indicators of mathematical communication skills used in this study were adapted according to NCTM (NCTM, 2000), as shown in Table 1.

<table>
<thead>
<tr>
<th>Indicator of Mathematical Communication Skill</th>
<th>Students’ Competence in Mathematical Communication</th>
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| 1. Communicate mathematical thinking to others in an orderly, logical, and clear manner | a. Communicate mathematical thinking logically to others.  
   b. Delivering mathematical thinking communication clearly to others. |
| 2. Organize and ensure mathematical thinking through communication. | a. Organize mathematical thinking through communication.  
   b. Consolidation of mathematical thinking through communication. |
| 3. Using mathematical language to express mathematical ideas correctly | a. Use mathematical language to express mathematical ideas correctly. |
| 4. Analyze and evaluate the mathematical thinking and strategies used by others | a. Analyze the mathematical thinking and strategies used by others.  
   b. Evaluating mathematical thinking and strategies used by others. |
Each indicator in Table 1 above has a sub-indicator as a competency in mathematical communication skills that will be achieved by students.

RESULTS AND DISCUSSION

From the results of the student's final examination in class B on the differential calculus course, the researcher analyzed question number one, which asked about the minimum-maximum. The score of 25 students was grouped into three categories of high, medium, and low. From each group, two students were taken. The results of student work with high categories are subjects ST1 and ST2, medium category SS2 and SS4, and low categories SR5 and SR6. The results of each subject's work are as follows.

Subject 1 (ST1)

Mathematical communication skills based on the results of the work of Subject 1 (ST1) are shown in Figure 2 below.

![Figure 2. The work from Subject 1 (ST1)](image)

with Good Mathematical Communication Skill

The result showed that the answers given by ST1 were all correct. ST1 has the ability to provide information on the questions that have been written, the completion steps are taken, the images asked, the mathematical models used to solve, and the answers according to what is known and asked. So the answers given showed that the subject has good mathematical abilities.
Subject 2 (ST2)

Mathematical communication skills based on the results of the work of Subject 2 (ST2) are shown in Figure 3 below. The information provided in the questions has been written correctly. The completion steps are taken in finding a stationary point were wrong because the right-hand side is not equal to zero. The drawing was also wrong because the image is not drawn correctly as asked. The mathematical model used to solve the problem was correct, writing down what is known and asked precisely, and the steps taken were appropriate. Writing down what is known and is asked with the wrong answer, namely in the area of origin problem \( x \neq -3 \), but in answer to the picture, there is a dot at \( x = -3 \).

![Figure 3. The Work from Subject 2 (ST2) with Good Mathematical Communication Skill](image)

Subject 3 (SS3)

Mathematical communication skills based on the results of the work of Subject 3 (SS3) are shown in Figure 4 below.
The information provided in the questions has been written correctly. There is nothing wrong with the complete steps that are carried out; the picture in question has been correctly made based on the steps of the previous question. The mathematical model used to solve it was also correct, writing down what is known and asked exactly what it should be. However, SS3 still has errors in the coordinates and graphical images that are not yet suitable. This is shown in Figure 5 (a) that there is an error, namely writing the
coordinates using curly brackets, so the meaning is different. Meanwhile, in Figure 5 (b) shows that the graphics are less refined; the subject is less able to make models accurately.

**Subject 4 (SS4)**

Mathematical communication skills based on the results of the work of Subject 4 (SS4) are shown in Figure 6 below.

![Figure 6](image)

(a)

(b)

**Figure 6. The Work from Subject 4 (SS4) with 4 Mistakes**

The information provided in the questions has been written correctly. However, SS4 made several mistakes as shown in Figure 6 (a) including: (1) The steps taken in finding a stationary point were wrong because the right side does not equal to zero; (2) The mathematical model used to solve the problem was correct, writing down what is known and asked correctly and the steps are taken were also appropriate, but the final answer was still wrong.

Furthermore, in Figure 6 (b) there are also errors including: (1) The depiction is not accurate because the image is not smoothed, which is not in accordance with the previous steps in the question; (2) Write down what is known and is asked with the wrong
answer, namely in the area of origin problem \( x \neq -3 \) but in answer to the picture, there is a dot at \( x = -3 \).

**Subject 5 (SR5)**

Mathematical communication skills based on the results of the work of Subject 5 (SR5) are shown in Figure 7 below.

Figure 7. The Work from Subject SR5 with Many Mistakes

Figure 7 (a) shows that SR5 made several mistakes, including (1) Unable to take steps to find a stationary point, an error occurred because the right-hand side does not equal to zero; (2) Not being able to find the minimum score because the subject unable to understand the information about the questions given; (3) Not being able to provide appropriate steps for solving, such as in finding the interval for the increase/decrease function; and (4) Write down what is known and is asked with the wrong answer, namely in the area of origin problem \( x \neq -3 \) but in answer to the picture there is a dot at \( x = -3 \).

Meanwhile, Figure 7 (b) shows that SR5 made an error including: (1) Not being able to determine the interval for the concave up or down function according to the available information; (2) Not able to depict precisely because the image is not refined which is not in accordance with the steps of the previous problem. The mathematical
model used to solve it is correct. Write down what is known and asked precisely and the steps taken are also appropriate; (3) Not able to write the information given correctly, such as the area of origin which is known -3 < x ≤ 4 to write -3 ≤ x ≤ 4 which means x = -3 is entered as the origin; and (4) Write down what is known and is asked with the wrong answer, namely in the area of origin problem x ≠ -3 but in answer to the picture there is a dot at x = -3.

**Subject 6 (SR6)**

Mathematical communication skills based on the results of the work of Subject 6 (SR6) are shown in Figure 8 below.

Figure 8. The Work from Subject SR6 with Four Mistakes

SR6 is able to write down the given information precisely, such as the function of origin. However, SR6 made a few mistakes. In Figure 8 (a) there is one error, namely the incorrect writing of the quadratic equation. In Figure 8 (b) there are three errors including: (1) Not being able to write a period, such as a back point not given brackets; (2) Not able to depict precisely because the image is not refined where it is not in accordance with the previous steps of the problem, the mathematical model used to solve correct, writing...
down what is known and asked precisely and the steps taken are also appropriate; and (3) Not able to answer according to what is known.

Based on the results of student work per given question, it can be analyzed as follows.

Critical Point

Students with high ability were able to answer correctly with no errors. While the moderate ability students (SS4) and low ability students (SR5 and SR6), they still made mistakes in working on the problem by writing \((3x-1)(x-1)\), which should be written \((3x-1)(x-1) = 0\). It can be said that they are less capable of completion steps. From the results of clarification through interviews, the following results were obtained:

P: "To find a critical point, are you able to understand?"
SS4: "Understood"

P: "To find a critical point, what steps did you take?"
SS4: "The critical point is obtained from the factor of the first derivative of the function and also the end of the boundary area of origin."

P: "In finding a stationary, why does point f" is made equal to zero?"
SS4: "To find the factor of the function."

P: "Why is your work of factoring are not equal to zero?"
SS4: "Oh, I forgot to wrote it down."

From the results of the interview, it can be seen that the students who make mistakes are actually able to understand the information well, able to take steps well, able to answer from what they know. The mistake is only in the step of factoring the minus plus equal to zero.

The results of this study, which are consistent with the results of research conducted by (Rohid et al., 2019, pp. 23–24) show that subject 1 writes symbols, notations, and principles in a very good way. This shows that subject 1 has excellent mathematical communication skills to represent mathematical ideas. Furthermore, other research that is in accordance with this study conducted by (Khoiriyah, 2016, p. 205) shows that subject 1 is able to use the right method but make mistakes in determining the interval.
Extreme Value

In determining the extreme score, all subjects answered correctly, except for the moderate student number SS4 who did not answer the minimum score and the SR5 from low ability student who had written the minimum score, which there shouldn't be any minimum score.

From this answer, it shows that the subject is less able to understand the information correctly and is unable to answer according to what is known. Below are the results of the interview with student SS4:

P: "How do you find the extreme value?"
SS4: "Substituting the critical point into the first function."
P: "Do you mean the first function is the function in the problem?"
SS4: "Yes"
P: "How to determine the maximum and minimum values?"
SS4: "The minimum value is the result of the substitution of the critical value to the first function whose value is low and enters the area of origin."
P: "Why is the minimum value missing or not written?"
SS4: "Because -3 does not include the area of origin."
P: "If it is not written, can the person who reads know that the minimum function value does not exist?"
SS4: "I didn't write it down because I thought it didn't exist."

For those who answered there was a minimum score; after clarification, information was obtained as written below:

P: "Why did you answer that the minimum score exists?"
SR5: "Because if f is continuous on the closed hose [a, b], then f reaches the maximum and minimum values."
P: "Do you understand that the questions given are not closed hoses?"
SR5: "Understand, the -3 is not closed because the sign is less than and 4 hoses are closed because the sign is less than or equal to"

From the interview, it shows that student SS4 is actually able to understand the information that is known and asked until the completion steps. The error that occurs is simply not writing the answer according to the question. For SR5 the error is in understanding a continuous function over a range that will reach the maximum or minimum value.

**The interval on Increasing and Decreasing Function**

Student SR5 made a mistake in determining the interval for the increasing and decreasing function. Other students are all able to solve this problem. SR5 cannot provide appropriate settlement steps to determine the interval of the increasing and decreasing function because this student cannot understand the information accurately.

The reason SR5 made this mistake after being clarified, the following results were obtained:

P: "How do you determine the interval in the increasing and decreasing function?"

SR5: "By knowing the value of the critical point."

P: "Why should we know the value of the critical point?"

SR5: "As a divider or interval divider into subintervals, so that in subintervals, we can determine positive or negative."

P: "What do you mean by positive?"

SR5: "The positive means it has a positive gradient, so the graph of the function goes up."

P: "How do you know positive or negative?"

SR5: "By entering one of the numbers in the interval as a test point."

P: "Why is there no such step in your answer?"

SR5: "I did, but I didn't write it down."

P: "Why is your answer in the increasing interval function is written -3≤x≤2 / 3?"
SR5: "Because the mark at the interval is positive."

P: "For the one with the answer -3≤x etc. where did it come from? Is it in line with the question?"

SR5: "In question -3 less than x, there is no equal sign with it, ma'am. So, my answer did not have an equal sign with it because -3 is not the area of origin."

From the results above, this student was actually able to solve the problem of finding the interval in the increasing and decreasing function. The student made a mistake due to a lack of accuracy in reading the question.

These errors found in this study were also found in the research conducted by (Jamilah & Septianawaty, 2017, p. 66), showing that there were three students who still did not correctly solve the questions because they were not thorough because they did not answer as asked.

**Interval in Concave Up Function or Concave Down Function**

Student SR5 is the only student who fails in determining the interval for increasing and decreasing functions. SR5 is not able to provide answers according to what is known and is not able to understand the information appropriately.

The interview result shows that the mistakes made are the same as in finding the interval of concave up and down because there is less accuracy in reading the questions. In the question looking for the interval on the concave up and down function, student ST2 gave an answer that shows good mathematical communication skills. The answers given are as follows:

P: "How do you find the interval in the concave up and down function?"

ST2: "The upward or downward concave function interval is obtained after calculating the second derivative of the known function which equals to zero, so that the x value is known which is the benchmark for finding the turning point."

P: "What if the second derivative you are looking for happens to be zero?"

ST2: "If the second derivative is zero?"

P: "If the second derivative in the function is zero, then there is no turning point in the function."
P: "What if you find both a positive derivative?"

ST2: "So what this function has is only the concave up function."

From the ST2 work results, which is included in the high-ability student and from the interview results, it can be understood that this subject has good mathematical communication skills because he can work on questions correctly.

**Turning point**

Some subjects are unable to determine turning points because they lack thoroughness in writing. Student SS3 has written the turning point by enclosing them in curly brackets. Meanwhile, SR6 writes the point without brackets. The reason given through the results of interviews with the two students was that after clarification through interviews with SR6, the following results were obtained:

P: "How do you find a turning point?"

SR6: "A turning point can be found if there is a change from concave upward to concave downward or the reverse."

P: "The turning point is the result in the form of a point?"

SR6: "yes."

P: "What if you write point use brackets, or braces or without brackets? Does it have the same meaning?"

SR6: "Use regular brackets because they have different meaning."

P: "Why don't you use regular brackets, if that's a point?"

SR6: "I forget, I should have given regular brackets, sorry I make a mistake."

**Graph of Function**

Some students couldn't perform well in drawing graphs of a function, such as those from moderate and low ability students, in which the graphic depiction was not refined so that the information obtained from the previous questions is not applied in drawing this graph. What information is known, such as the area of origin, is also not well understood. It is proved in the graph depiction of the function. They still drawing a point (-3, -47).
These errors found in this study are also found in the results of research conducted by (Khoiriyah, 2016) showing that student number 2 in drawing graphic images does not apply the right steps, and as a result, the image is not clear and difficult to understand.

From the interview with ST1 from high ability students, the results below is obtained:

P: "To draw a graphic, is it related to the previous question?"

ST1: "Yes, there is."

P: "Which part?"

ST1: "About the critical point."

P: "Are other parts unnecessary?"

ST1: "Only the critical point."

While the answer from the less perfect in drawing by student SR6:

P: "How do you draw the graphics?"

SR6: "It's a bit difficult because there are many points sometimes forget which ones should be included in the area of origin and which are not included in the area of origin."

P: "What steps did you do in drawing the graphics?"

SR6: "Writing down the coordinate points of the existing extreme values."

P: "Does drawing have anything to do with the previous questions? Which part?"

SR6: "Yes, at an extreme value because to draw a graph, we need an extreme value as a coordinate."

P: "Is that all you have to know?"

SR6: "As far as I know, that's all."

P: "Why is the image not refined?"
SR6: "We are sorry when I was not thorough so that the image had not been refined and there were points that should not have been included; instead they were included in the area of origin."

The answers given show that students' mathematical communication skills related to drawing graphics are still lacking. Drawing graphics should not only be limited to knowing critical points and extreme values. But an interval in the increasing and decreasing function is also needed, an interval in the concave up and down function, a turning point so that the image is exactly in accordance with the information.

From the students' work and interviews, it can be said that students with high ability have good communication skills because there is no mistake within their work, as, for students with moderate ability, they still make some mistakes due to their lack of accuracy. Meanwhile, those with low ability of mathematical communication skills need to improve their ability, especially in drawing a graph of functions.

The results of this study are following the results of research conducted by (Paridjo & Waluya, 2017, pp. 65–66), showing that most students are able to analyze and evaluate problems well, but there are some students who make mistakes, which indicate that students these have weak communication skills, so they need to improve their ability.

CONCLUSION

Based on the results and discussion about six students in class B of Mathematics Education Program, Ahmad Dahlan University in the academic year 2019/2020, it can be concluded that mathematical communication skills in the minimum-maximum material are as follows: (1) restating information is done appropriately, (2) Writing down ideas or steps to solve the problem clearly and precisely, some are not thorough, (3) Representing information using graphic images is still wrong and not accurate, and (4) In writing answers according to what is known and asked using mathematical formulas, there are some students who still make mistakes because they are less precise.
REFERENCES


